Bellcomm

955 L'Enfant Plaza North, S.W.
Washington, D. C. 20024

B72 01011

date: January 25, 1972

to: Distribution

from: W. Levidow

subject: Control of Skylab CMG Momentum Level by Momentum Dump Laws - Case 620

ABSTRACT

The Skylab CMG momentum level must be controlled in order to constrain the momentum variation within saturation limits.

Two laws are currently being considered for determining the momentum to be dumped each orbit. The first, which uses sampled CMG momentum values from the current orbit only, is not likely to cause convergence of the momentum level to a given bias level. The difference depends upon how well the orbital bias momentum is estimated from the sampling and how accurately the commanded dump momentum is actually dumped.

The second, which uses sampled CMG momentum from the previous as well as the current orbit, does cause convergence of the momentum level to a given bias level. It is the better choice unless it can be shown that the difference resulting from using the first law can be adequately predicted.

(NASA-CR-126273) CONTROL OF SKYLAB CMG MOMENTUM LEVEL BY MOMENTUM DUMP LAWS (Bellcomm, Inc.) 10 p

N79-72429

Unclas
00/18 12202

Bellcomm

955 L'Enfant Plaza North, S.W.
Washington, D. C. 20024

B72 01011

date: January 25, 1972

to: Distribution

from: W. Levidow

subject: Control of Skylab CMG Momentum Level by Momentum Dump Laws - Case 620

MEMORANDUM FOR FILE

Introduction

To constrain the Skylab CMG momentum within saturation limits, excess momentum is normally dumped each orbit by vehicle maneuvers on the dark side of the earth. The commanded momentum-to-be-dumped is determined just prior to the maneuvers by a dump law whose inputs are sampled values of CMG momentum at selected positions on the light side portion of the orbit.

Two dump laws are currently being considered. The first, which might be called a Single-Orbit dump law, uses sampled momentum values only from the current orbit.* The second, which might be called an N-Orbit dump law, uses sampled momentum values from the previous as well as the current orbit.

This memorandum deals with the effectiveness of each dump law in controlling the CMG momentum.

Symbols

Below is a list of symbols used in this memorandum. Some are described more fully in the text.

$\underline{H}_O$ = Orbital bias momentum

$\underline{H}_K$ = Estimated value of $\underline{H}_O$ based on sampling

K_O = Estimate factor, where $\underline{H}_K = K_O \underline{H}_O$

$\underline{H}_B$ = Bias momentum level

---

*For the purpose of this discussion, an orbit is assumed to start at the termination of the previous orbit's dump maneuvers.

$\underline{H}_A$ = Deviation of the CMG actual momentum level from the bias momentum level $\underline{H}_B$

$\underline{(\Sigma H)}$ = A summation of previous orbit's deviations

$\underline{H}_D$ = Commanded dump maneuver momentum change

K_D = Dump factor

$K_D\underline{H}_D$ = Actual dump maneuver momentum change

C_1, C_2, C_3 = Dump law gains

Momentum Sampling

As indicated in Fig. 1a, CMG momentum sampling occurs at positions 2 and 6 ($\pm 90°$ from dump midnight) and positions 3 and 5 ($\pm 135°$ from dump midnight). The corresponding momenta are called $\underline{H}_2$, $\underline{H}_3$, $\underline{H}_5$, and $\underline{H}_6$. A plot of typical CMG Y axis momentum variation during sampling is shown in Fig. 1b. The variation is due mainly to gravity gradient torque. The difference between $\underline{H}_2$ and $\underline{H}_6$ (or $\underline{H}_3$ and $\underline{H}_5$) is due to bias components of the gravity gradient or other disturbance torques.

The orbital bias momentum, $\underline{H}_O$, is defined as the change in CMG momentum over one orbit if dump maneuvers were not executed. Its estimate, $\underline{H}_K$, is calculated by

$$\underline{H}_K = 2(\underline{H}_6 - \underline{H}_2) \tag{1}$$

$\underline{H}_K$ may differ from $\underline{H}_O$ because, while the estimate is based on half orbit sampling, both halves may not be identical (aerodynamic torque, for example is not symmetrical with respect to the sampling positions 2 and 6).

The CMG momentum level is defined as the average of $\underline{H}_3$ and $\underline{H}_5$. This level should be controlled in order to hold the CMG momentum variation centered within the saturation limits. The optimal level depends upon the angle between the orbital plane and the Z axis and upon the required dump maneuvers, for both affect the CMG momentum variation with respect to the vehicle axes. The level may also require shifting to a favorable value in preparation for Z-local-vertical maneuvers which require large CMG momentum swings.

The Mission Control Center will determine the optimal level by monitoring and plotting the CMG momentum variations during flight. The CMG level will then be controlled by the input of a bias level, $\underline{H}_B$, into the ATM Digital Computer. The purpose of the dump law is to cause the actual level to converge to the bias level.

The deviation $\underline{H}_A$ of the actual CMG momentum level from the bias level is calculated by

$$\underline{H}_A = (\underline{H}_3 + \underline{H}_5)/2 - \underline{H}_B \tag{2}$$

The Y axis components of $\underline{H}_K$ and $\underline{H}_A$ are indicated in Fig. 1b.

$\underline{H}_K$ and $\underline{H}_A$ are inputs to both the Single-Orbit and the N-Orbit dump laws.

Momentum Dump Laws

Both laws generate a commanded dump maneuver momentum change $\underline{H}_{D_n}$, where n refers to the nth orbit.

For the Single-Orbit law,

$$\underline{H}_{D_n}{}^{*} = -\underline{H}_{A_n} - \underline{H}_{K_n} \tag{3}$$

For the N-Orbit law,

$$\underline{H}_{D_n}{}^{*} = \underline{(\Sigma H)}_n - \underline{H}_{K_n} \tag{4}$$

where

$$\underline{(\Sigma H)}_n = \underline{(\Sigma H)}_{n-1} - \underline{H}_{A_n} + .75\underline{H}_{A_{n-1}} \tag{5}$$

It is desireable for each law that $\underline{H}_{A_n}$ converge to 0 in the steady state; that is, the momentum level (average of $\underline{H}_3$ and $\underline{H}_5$) converge to the given bias level $\underline{H}_B$. How well this can be accomplished with each law will now be developed.

---

*ATMDC Program Definition Document Part I, Section 10 Revision 9.

Single-Orbit Dump Law

The Single Orbit law may be written as

$$\underline{H}_{D_n} = -C_1\,\underline{H}_{A_n} - \underline{H}_{K_n} \tag{6}$$

$$= -C_1\,\underline{H}_{A_n} - K_O\,\underline{H}_{O_n} \tag{7}$$

For both dump laws,

$$\underline{H}_{A_{n+1}} = \underline{H}_{A_n} + \underline{H}_{O_n} + K_D\,\underline{H}_{D_n} \tag{8}$$

where K_D accounts for the fact that due to approximations used in calculating the dump maneuvers, the actual momentum dumped may not exactly equal the commanded momentum dumped.

Taking z-transforms of Eqs. 7 and 8 and assuming $\underline{H}_O$ to be constant over several orbits,

$$\underline{h}_D = -C_1\,\underline{h}_A - K_O\,\underline{H}_O\,z/(z-1) \tag{9}$$

$$z\,\underline{h}_A = \underline{h}_A + \underline{H}_O\,z/(z-1) + K_D\,\underline{h}_D \tag{10}$$

where $\underline{h}$ denotes the z-transform of $\underline{H}$.

Combining Eqs. 9 and 10,

$$\underline{h}_A = \frac{\underline{H}_O\,z(1-K_D K_O)}{(z-1)[z-(1-C_1 K_D)]} \tag{11}$$

By the final value theorem,

$$\lim_{n\to\infty} f(n) = \lim_{z\to 1} \{(z-1)\,F(z)\} \tag{12}$$

provided that the system is stable (F(z) has no poles outside the unit circle and no double or higher order poles on the unit circle).

Therefore,

$$\lim_{n\to\infty} \underline{H}_{A_n} = \frac{\underline{H}_O}{C_1}\left(\frac{1}{K_D} - K_O\right) \tag{13}$$

Provided that

$$|1-C_1K_D| \leq 1.0 \tag{14}$$

or

$$0 \leq C_1K_D \leq 2.0 \tag{15}$$

With C_1 chosen equal to 1.0 (Eq. 3), the system is stable and Eq. 13 is valid even with large inaccuracies ($0 \leq K_D \leq 2.0$) in the dump maneuvers.

From Eq. 13, $\underline{H}_{A_n}$ converges to $\underline{0}$ only if $K_DK_O = 1.0$, an unlikely coincidence. That $\underline{H}_{A_n}$ is not likely to converge to $\underline{0}$ could have been anticipated by observing (Eq. 8) that if $\underline{H}_{A_n} = \underline{0}$ the first orbit, then exactly $\underline{H}_{O_n}$ must be dumped each orbit to maintain $\underline{H}_{A_n} = 0$. But $\underline{H}_{O_n}$ will not necessarily be dumped because $\underline{H}_{K_n}$ (Eq. 6) is only an estimate of $\underline{H}_{O_n}$ and furthermore, the actual momentum dumped doesn't necessarily equal that commanded.

Hence on subsequent orbits $\underline{H}_{A_n}$ ultimately builds up to a value, Eq. 13, which when substituted in Eq. 6 provides the proper command for $\underline{H}_{O_n}$ to be dumped.

N-Orbit Dump Law

The N-Orbit law may be written as

$$\underline{H}_{D_n} = \underline{(\Sigma H)}_n - \underline{H}_{K_n} \tag{16}$$

$$= \underline{(\Sigma H)}_n - K_O \, \underline{H}_{O_n} \tag{17}$$

where

$$\underline{(\Sigma H)}_{n+1} = \underline{(\Sigma H)}_n - C_2 \underline{H}_{A_{n+1}} + C_3 \underline{H}_{A_n} \tag{18}$$

Taking z-transforms of Eqs. 17 and 18,

$$\underline{h}_D = \underline{(\Sigma h)} - K_O \, \underline{H}_O \, z/(z-1) \tag{19}$$

$$z \, \underline{(\Sigma h)} = \underline{(\Sigma h)} - z \, C_2 \underline{h}_A + C_3 \underline{h}_A \tag{20}$$

Combining Eqs. 8, 19, and 20

$$\underline{h}_A = \frac{\underline{H}_O \, z \, (1-K_D K_O)}{z^2 + z(C_2 K_D - 2) + (1 - C_3 K_D)} \tag{21}$$

$$\underline{h}_D = \underline{h}_A \frac{(z-1)}{K_D} - \frac{\underline{H}_O \, z}{K_D (z-1)} \tag{22}$$

$$\underline{(\Sigma h)} = \frac{\underline{h}_A (z-1)}{K_D} + \frac{\underline{H}_O \, z (K_O - \frac{1}{K_D})}{(z-1)} \tag{23}$$

By the final value theorem

$$\lim_{n \to \infty} \underline{H}_{A_n} = 0 \tag{24}$$

$$\lim_{n \to \infty} \underline{H}_{D_n} = -\underline{H}_O / K_D \tag{25}$$

$$\lim_{n \to \infty} \underline{(\Sigma H)}_n = \underline{H}_O (K_O - \frac{1}{K_D}) \tag{26}$$

provided that

$$\left| 1 - \frac{C_2 K_D}{2} \pm \left(\frac{(C_2 K_D)^2}{4} + K_D (C_3 - C_2) \right)^{\frac{1}{2}} \right| < 1.0 \qquad (27)$$

For $C_2 = 1.0$ and $C_3 = 0.75$ (values in Eq. 5), the system is stable and Eqs. 24, 25, and 26 are valid for $0 < K_D < 2.25$. Hence convergence of $\underline{H}_{A_n}$ to $\underline{O}$ tolerates large inaccuracies in the dump maneuvers. Also, convergence is independent of K_O, that is, how well the orbital bias momentum is estimated.

Greater insight into the operation of the N-Orbit law can be gained if Eq. 18 is rewritten as

$$\begin{aligned} \underline{(\Sigma H)}_n = & -C_2 \underline{H}_{A_1} \\ & -C_2 \underline{H}_{A_2} + C_3 \underline{H}_{A_1} \\ & -C_2 \underline{H}_{A_3} + C_3 \underline{H}_{A_2} \\ & \text{--------------} \\ & - C_2 \underline{H}_{A_n} + C_3 \underline{H}_{A_{n-1}} \\ = & (C_3 - C_2) \sum_{r=1}^{n-1} \underline{H}_{A_r} - C_2 \underline{H}_{A_n} \end{aligned} \qquad (28)$$

By Eq. 16

$$\underline{H}_{D_n} = (C_3 - C_2) \sum_{r=1}^{n-1} \underline{H}_{A_r} - C_2 \underline{H}_{A_n} - \underline{H}_{K_n} \qquad (29)$$

The first term on the right of Eq. 29 acts as a dummy variable which in steady state builds up to a value which provides the proper command $\underline{H}_{D_n}$ for dumping $\underline{H}_{O_n}$. A comparison of Eq. 26 and Eq. 13 shows that it converges to the same magnitude as does $C_1\underline{H}_{A_n}$ of the Single-Orbit law. This allows $\underline{H}_{A_n}$ of the N-Orbit law to converge to $\underline{O}$.

Conclusions

The N-Orbit dump law, in which the commanded dump momentum is a function of sampled CMG momentum on previous as well as the current orbit, causes the CMG momentum level to converge to the input bias level $\underline{H}_B$. Hence the CMG level can be controlled accurately. However, the N-Orbit law requires slightly greater computer capacity than the Single-Orbit law.

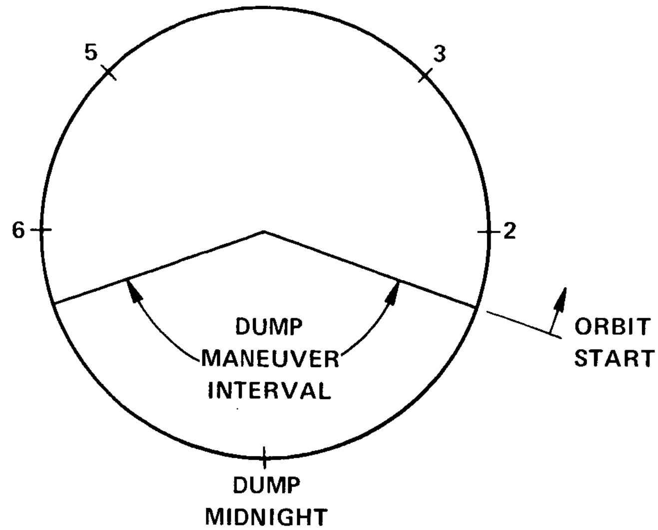
The Single-Orbit law, which depends on sampled CMG momentum from only the current orbit, is not likely to cause convergence of the momentum level to $\underline{H}_B$. The deviation depends upon how well the orbital bias momentum is estimated from half orbit sampling and how faithfully the commanded dump momentum is actually dumped.

If the level converges to $\underline{H}_B$, as in the N-Orbit law, then $\underline{H}_B$ can be used directly to control the level to a desired value. If not, as in the Single-Orbit law, then the steady state deviation must be predicted and allowed for in applying $\underline{H}_B$. The real time deviation will become apparent from the Mission Control Center CMG momentum plots.

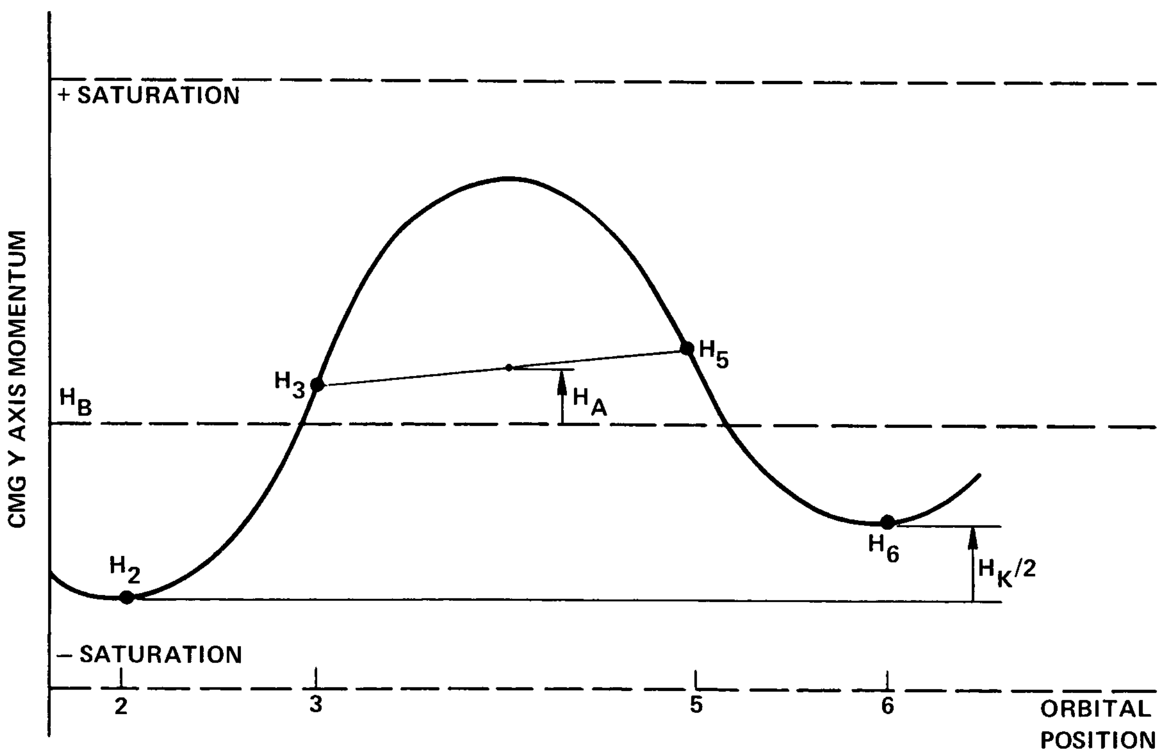
If pre-flight computer simulations indicate that the deviation is not erratic (that is, K_D and K_O do not change abruptly) and can be predicted well with changing vehicle attitude and disturbance torque, then the Single-Orbit law will suffice. If not, the N-Orbit law is required.

W. Levidow

1022-WL-mef W. Levidow



1a - SAMPLING POSITIONS



1b - TYPICAL CMG Y AXIS MOMENTUM VARIATION

FIGURE 1 - CMG MOMENTUM SAMPLING